



Predictability of multifractals processes and geophysics

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Time complexity is associated with sensitive dependence to initial conditions and severe intrinsic predictability limits [1], in particular the 'butterfly effect' paradigm: an exponential error growth and a corresponding characteristic predictability time. This was believed to be the universal long-time asymptotic predictability limits of complex systems.

However, turbulence and geophysics are complex both in space and time and have rather different predictability limits: a limited uncertainty on initial and/or boundary conditions over a given subrange of time and space scales grows across the scales and there is no characteristic predictability time. We showed [2] that complexity in space implies strong limitations on the applicability of the Multiplicative Ergodic Theorem (MET, [3]) and of the Liouville equation [4].

The relative symmetry between time and space yields scaling (i.e. power-law) decays of the predictability, as confirmed by homogeneous turbulence phenomenology and statistical closure models [5, 6]. Unfortunately, the quasi-normal framework of these models prevents them from dealing with intermittency: the "bursts" of the energy fluxes through scales, as well as those of information loss [7].

We show that multifractals [8] offer a very convenient framework to quantify the predictability of space-time complex systems with the help of an infinite hierarchy of exponents. Furthermore, this hierarchy is defined in a straightforward manner for a large class of space-time multifractal processes. We also show that the corresponding scaling function can be used to empirically quantify the predictability of geosystems, as well the performance of forecast procedures. In particular, this readily explains the recent empirical evidence that stochastic subgrid parametrizations do better than

deterministic ones [9, 10].

These results will be illustrated with the help of various numerical simulations,

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