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Universal precursor of the earthquakes. Experience of the retrospective prediction

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Introduction

Theoretical and the experimental research of geomechanical oscillatory processes allow to study the influence of geophysical parameters on instabilities, such as the earthquake process. Unstable phenomena like earthquakes can occur in geomechanical systems which have the unstable equilibrium state at some set of critical geophysical parameters [1]. There are two fields of geophysical parameters, corresponding to stable and a unstable states. According to a theorem formulated in [1,2], there will be eigen vibratory movements of the geosystem for the stable field, and the frequencies will tend to zero as the system approaches unstable equilibrium (exact before earthquake). The critical wave length of vibrations at zero frequency remain finite, however, and characterize the size of the instability i.e. the dispersion wave motion before catastrophe event (for example earthquake) differs from usual dispersion $\omega = ck$. A change of eigen frequencies affects the seismoacoustic emission spectrum in the area surrounding an impending earthquake. This change indicates that the geomechanical system is close to an unstable threshold, and the critical wave length determines the energy and space dimensions of the developing instability, or earthquake.

The important term "unstable equilibrium" implies such state of a system, when two (and may be more) forces applied at rock massif balance each other. Such equilibrium could exist at certain values of elasticity and density of the geomechanical systems. These two main forces are the tectonic (in the end gravity) force and the effective friction force along fault. Equilibrium conditions can be violated if the friction force will decrease relative to the gravity (tectonic) force due to changing of the system's elastoplastic parameters values. Therefore such equilibrium is unstable. In that case the phenomena which will cause the slip instability start to develop. The larger difference between tectonic (gravity) force and friction one is the faster slip instability process should be.

The goal of our study is the analysis of geomechanical events, which should accompany earthquake preparation at its evolution towards the unstable equilibrium i.e. directly up to the catastrophic threshold when rapid failure occurs. There is a possibility to find the prognostic phenomena indicating such approach to the unstable equilibrium state and by that to predict time and scale of the earthquake. It can be done with a help of the general theorem, concerning the unstable equilibrium state systems.

Universal precursor theorem

The study of the universal earthquake precursors is based on a general theorem concerning the behavior of geomechanical systems as they approach instability (catastrophes threshold):

If the system has the state of the unstable equilibrium at some set of the critical parameters describing it and this set separates areas of the parameters values relevant to a stable state and unstable state of the system then in stable parameters area external load would cause eigen oscillation of the system with frequencies, which will tend to zero if the system approaches unstable equilibrium at the finite critical wavelengths [1,2]. Firstly the basic idea of this theorem was formulated in [3].

According to the theorem any catastrophe (earthquake) should be preceded by slow oscillation of some parameters describing the system which can have the state of the unstable equilibrium. Frequency of these eigen wave motions tends to zero as approaching an instability threshold.

It is possible to give some examples of preliminary behaviour before catastrophe threshold for such systems: alternating vertical and horizontal tectonic motions before orogeny, slow movements of the daylight surface, registered prior to earthquakes by the geodesic methods at a former Garm's polygon (Tadjikistan) of the Institute of Physics of the Earth, the onset of the slip instability along faults [4], simulating processes of earthquake, rock burst and fracture development which are preceded by the waves propagating along faults – trapped waves [5], slow wave motions before earthquakes in the Caucasus, Turkey, Balkans (the Black Sea region) where seiche oscillations have been registered as a result of these wave motion [6].

All these examples are characterised by the presence of the unstable equilibrium state at certain values of system parameters. Genesis of such equilibrium is equality of tectonic forces and friction forces of rock formation. The processes of the fracture's activation (or of the formation of new rupture) and consequent movement along it at a catastrophic stage take place in case of earthquakes. Therefore it is useful to note observed phenomena preceding such a catastrophic event as an earthquake.

Observable phenomenon of the low seismicity splash and its disappearance directly before relatively strong earthquake can be explained by the frequency decreasing of the waves while approaching of the geophysical parameters values of the future earthquake source to the critical values relevant to a catastrophic threshold. Since seismological equipment can register only those oscillations, which frequency fall within registration range, consequent low frequency oscillations could be missed by seismic network, but can be recorded by broadband instruments. Besides in the system with dissipation it is possible actual silent directly before the earthquake when any disturbance attenuates, but more and more slowly as approaching a catastrophic threshold [1]. In any case these two stages (at first oscillatory and then aperiodic) of the prognostic precursor should present always, as they are essential parts of the earthquake source evolution towards the instability threshold.

Moreover, the considered wave deformation process preceding catastrophe, can initiate other precursor which have different physical nature: increasing of the radon abundance and changing of the water level in boreholes as the consequence of the crust permeability increasing at its periodic deformations, or the electrical and electromagnetic phenomena in atmosphere and ionosphere [7] which can appear due to functioning of the electrohydrodynamic and electroelastic mechanism of the electric field generation and, hence, of the electromagnetic radiation [8]. The same can be exemplified by the above mentioned registration of the seiche oscillations in Black Sea [6], which directly correspond to the slow waves motion. Seiche oscillations are generated due to resonance phenomenon when the changing frequency of the pre-earthquake slow waves fall within the seiches frequency range. We can also note that the same effect of the seismoacoustic background noise frequency decreasing is observed before the rock bursts in mines and during laboratory experiments of fault formation [9]. Thus, random perturbations will grow as the system will develop from its initial state to a new one, due to energy transfer at a catastrophic stage of the process.

Theorem substantiation

The formulation of the theorem is done within the framework of linearized stability theory. It means, that nonlinear system is studied with perturbation theory, associated with linearized equations in partial derivatives. The linearized stability theory states, that if there is any solution of the linearized equations which grow with time, then the basic state we are studying is unstable. The proof of our theorem is simple for the case of conservative (nondissipative) systems. We shall give it here in order to explain the physical sense of the problem. Equilibrium of a system means, that solving the equations of elasticity (generally the linearized equations in partial derivatives), with some boundary conditions, we will get dispersion equation $\Delta(k,q) = 0$, where k is wave number and q is one or more dimensionless parameters (depending on elastic and density physical characteristics of the system). The solution of this equation gives the critical wave number $k_c = 2\pi/L_c$ at some q_c (L_c is wavelength) and the subscript "c" refers to the critical value. For the dynamic (time-dependent) problem with the same boundary conditions, the characteristic (dispersion) equation is $f(\omega,k,q) = 0$, where ω is angular frequency. In the limit $\omega \to 0$, must be $f(\omega,k,q) \to \Delta(k,q)$. In this limit, we can carry out the expansion of the dispersion equation $f(\omega,k,q) = 0$ to order ω^2 (as the system is conservative). Then to a first approximation, we shall obtain:

$$f(\omega,k,q) \approx \Delta(k,q) + (\partial f/\partial\omega^2)_{\omega=0}\,\omega^2 = 0 \tag{1}$$

From here, a key relation at once follows:

$$\omega^2 \approx -\Delta(k,q)/(\partial f/\partial\omega^2)_{\omega=0} \tag{2}$$

In the neighborhood of $k_c = 2\pi/L_c$, the function $\Delta(k,q)$ will have different sign on either side of k_c. Consequently on one side of k_c, ω will be imaginary, giving an exponentially growing solution, and on the another side, ω is real, giving an oscillatory solution. Thus for a conservative system, a simple but general mathematical analysis shows, that slow wave motion precedes and forecasts instability and shows what the catastrophic phenomena should be. The behavior of dissipative systems is more complicated as the equation for frequency will depend also on derivatives $(\partial f/\partial\omega^2)_{\omega=0}$. In this case, frequency appears complex and the behavior of the real and imaginary parts of frequency becomes more complicated than for systems without dissipation [1].

In mathematical and physical literature, the study of solutions of linearized equations in partial derivatives, describing the behavior of perturbations of nonlinear processes in geomechanical systems, is conducted in order to understand the conditions of instability. In this proposed work, we will concentrate our study on the frequency behavior of a system near the threshold of the instability, to elaborate methods of the earthquake forecasting. Such approach is effective and was not systematically applied before as we know.

The onset of slip instability

It is accepted by majority geophysicists, that earthquakes are connected close with development of the slip instability along faults. Consequently it is desirable to study such instability using well known Lyapunov's approach which essence is to study the solution of the linearized equations. These equations describe wave motion disturbances of the basic system state. And the basic system behavior is described by the corresponding nonlinear system. The main point of the Lyapunov's method is statement that if it will be found any solution of the linearezed equation which grow with time then basic state must be considered unstable. Namely our universal precursor theorem is closely connected with Lyapunov's approach as it was marked above.

So we will study the propagation of elastic waves along fault with velocity and displacement dependent friction as the perturbation of uniform slip speed. The fault separates two identical elastic half spaces. The mathematical statement of the problem is formulated as follows. Wave motion of the elastic media is described by well known elastic equations for displacement u:

$$\rho\frac{\partial^2 u}{\partial t^2} + \mu \mathrm{curl}\,\mathrm{curl} u - (\lambda + 2\mu)\nabla \mathrm{div} u = 0 \tag{3}$$

where λ and μ are Lame parameters. The stress-strain relationship is taken in Hooke's Law form:

$$\sigma_{ik} = \lambda \mathrm{div} u \delta_{ik} + \mu\left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i}\right) \tag{4}$$

We consider two-dimensional plane strain problem with the x - axis along and y - axis directed perpendicular to the fault. Dependance on the third coordinate z we disregard. To formulate finally mathematical problem with equations (3) and (4) we set boundary conditions as follows: 1. Exponential attenuation of the solution perpendicular to the fault direction; 2. Continuity of the fault-normal displacements and continuity of normal and shear stresses across fault; 3. Shear stress must satisfy the linearized friction low with discontinuity of the longitudinal displacement across fault:

$$\sigma_{xy}^{(1)}(x,0,t) = \chi P\left(u_x^{(1)}(x,0,t) - u_x^{(2)}(x,0,t)\right) \tag{5}$$

where indexes (1) and (2) mark half-spaces 1 and 2 respectively, P is pressure on the fault and χ is functional derivative $\chi = \delta f/\delta u$ of the frictional coefficient f: $\chi(x,v,t) = (a + i\omega b)u_x$ where $a = (\partial f/\partial x)_0$ and $b = (\partial f/\partial v)_0$ (the subscript "0" indicate derivative in $x = x_0$, $v = v_0$) at that $u_x \ll x_0$ and $\partial u_x/\partial t \ll v_0$.

For propagation of waves along the fault we specify dependance of all displacements periodically on time and x coordinate:

$$u^{(i)}(x, y, t) = \overline{u}^{(i)}(y) \exp\{i(\omega t - kx)\} \tag{6}$$

where $k = 2\pi/\lambda$ is wave number. For media (1) the general solution of equations (3) satisfying the boundary condition 1 is:

$$u_y^{(1)} = \left[\frac{C}{2}(e^{-\alpha_1 y} + e^{-\alpha_2 y}) + \frac{Dk}{(\alpha_2 - \alpha_1)}(e^{-\alpha_1 y} - e^{-\alpha_2 y})\right] \exp\{i(\omega t - kx)\} \tag{7}$$

$$\begin{aligned} u_x^{(1)} = i \Big[& \frac{A}{2}\left(\frac{\alpha_1}{k}e^{-\alpha_1 y} + \frac{k}{\alpha_2}e^{-\alpha_2 y}\right) \\ & + \frac{Bk}{(\alpha_2 - \alpha_1)}\left(\frac{\alpha_1}{k}e^{-\alpha_1 y} - \frac{k}{\alpha_2}e^{-\alpha_2 y}\right)\Big] \exp\{i(\omega t - kx)\} \end{aligned} \tag{8}$$

where $\alpha_1 = k\sqrt{1 - c^2/c_s^2}$, $\alpha_2 = k\sqrt{1 - c^2/c_p^2}$, and $c = \omega/k$, $c_s^2 = \mu/\rho$, $c_p^2 = (\lambda + 2\mu)/\rho$. For media (2) corresponding solution is:

$$u_y^{(2)} = \left[\frac{C}{2}(e^{\alpha_1 y} + e^{\alpha_2 y}) + \frac{Dk}{(\alpha_2 - \alpha_1)}(e^{\alpha_1 y} - e^{\alpha_2 y})\right] \exp\{i(\omega t - kx)\} \tag{9}$$

$$\begin{aligned} u_x^{(2)} = -i \Big[& \frac{A}{2}\left(\frac{\alpha_1}{k}e^{\alpha_1 y} + \frac{k}{\alpha_2}e^{\alpha_2 y}\right) \\ & + \frac{Bk}{(\alpha_2 - \alpha_1)}\left(\frac{\alpha_1}{k}e^{\alpha_1 y} - \frac{k}{\alpha_2}e^{\alpha_2 y}\right)\Big] \exp\{i(\omega t - kx)\} \end{aligned} \tag{10}$$

Equations (7), (8) and (9), (10) employ the convention that the y-axis is directed into media (1). This convention is maintained throughout. Constants A, B, C, and D are determined from boundary conditions 2 and 3 at the surface separating the two elastic half-spaces.

The dispersion equations relating frequency ω and wave number k of the propagating wave are obtained by a standard method, substituting the solution (7), (8) and (9), (10) into the boundary conditions and using Hooke's Law (4):

$$\sqrt{1 - \frac{c^2}{c_s^2}}\sqrt{1 - \frac{c^2}{c_p^2}} - \left(1 - \frac{c^2}{2c_s^2}\right)^2 = -q\frac{c^2}{2c_s^2}\sqrt{1 - \frac{c^2}{c_s^2}} \tag{11}$$

where $c = \omega/k$ – phase velocity and $q = \chi P/k\mu$. If the right side of equation (11) equals zero, we obtain the exact equation for a Rayleigh wave. For other case, the speed of wave propagation along fault depends on the frictional characteristics of the surface. This introduces displacement dependence that is specified by the dimensionless parameter $q = \chi P/k\mu$, where χ is slope of the friction coefficient f on relative displacement.

It is difficult to find the dependence of the speed c on friction parameters unless the analytical dependence of the friction coefficient is specified. In the general case, such an analytical expression may by impossible to obtain as it strongly depends on changing physical and chemical conditions of the frictional surfaces. But some conclusions can be made for the limiting case of low wave speed, when $c/c_s \ll 1$. This case is both interesting and easy to study. For this purpose we decompose the left side of (11) keeping terms up to second power of c/c_s or c/c_p. We obtain:

$$(\lambda + \lambda_2)\frac{c^2}{c_s^2} + i\frac{4\pi b c_s}{|a|} + 4(\lambda + \lambda_1) = 0 \tag{12}$$

where λ – wave length of the wave disturbance, propagating along fault,

$$\lambda_1 = \frac{2\pi\mu\left(c_p^2 - c_s^2\right)}{c_p^2 Pa} \quad \text{and} \quad \lambda_2 = \frac{2\pi\mu\left[1 - \left(c_p^2 - c_s^2\right)^2/2c_p^4\right]}{Pa}$$

two characteristic wave length which value is determined by inverse slope a of the friction coefficient in dependence on displacement.

Model problem studying here describes wave propagating along fault with amplitude which exponentially attenuates perpendicular to the fault direction. Therefore such wave has name trapped wave [5]. As we mentioned its phase velocity depends on the friction characteristics of the fault surfaces and the friction history (value a).

It is important that in the limit $\omega \to 0$ (or $c \to 0$) from (12) follows $\lambda = -\lambda_1$, i.e. there exist stationary solution problem we are studying with characteristic wave length disturbance $|\lambda_1|$. It means that the conditions of the universal precursor theorem are fulfill. Then we can state taking into account this theorem that uniform slip along fault is unstable and takes place stick-slip motion if $a < 0$ i.e. friction depends on displacement inversely. Each episode of the instability correspondents to the catastrophic transition (earthquake) from one evolution process with uniform slip to another one. During evolution the friction coefficient is changing (for example decreasing due to $a < 0$) preparing our system, consisting of two elastic half-space which divided by fault, to the new catastrophic stage.

Conclusions
Experience of the retrospective prediction

To predict the earthquake means to find out the phenomena which accompany the evolutional process of the earthquake preparation. The formulated universal precursor theorem [1,2] let us such possibility. Namely we have to find the low frequency shift of the seismicity spectra which must exists before earthquake. As it was shown in [3] such kind of precursor is short term precursor, i.e. the most important. The rate of the seismic spectra shift can show us the time of catastrophe- earthquake. And the critical wave length can clarify the dimension of the earthquake source (in the end the earthquake magnitude). This critical wave length could be found using unusual dispersion of the wave preceding to earthquake. Of course to work up corresponding spectra is rather difficult problem. But it was successfully and carefully done by G.A. Sobolev [10,11] on the base some physics and mathematics assumption. It was shown that seismic spectra maximum shifts before Kamchatka strong earthquakes in low frequency range in accordance with universal precursor theorem [1,4]. Another experiment of retrospective prediction was made in Sevastopol using data of measurements by laser strainmeter. It was found that before Romanian earthquake (10.27.2004, $M = 5.9$) took place shift of the Black Sea seiche oscillations in low frequency range (private information by V. Nasonkin and O. Boborikina). As it was mentioned before Black Sea seiche oscillations are generated by slow wave motion before earthquake in the Black Sea region. It is necessary to emphasize that registration of the low frequency oscillations, which are precede to the earthquake can be done more successively by broad range equipment, as it was demonstrated in K. Kasahara's book [12]. There is low frequency on the presented by him seismogram, registered due to Alaska earthquake (03.28.1964, $M = 8.5$).

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