



Bi-directional water waves and integrable high order KdV equations

G.I. Burde

Jacob Blaustein Institute for Desert Research, Ben-Gurion University (E-mail: georg@bgu.ac.il/ Fax: 972-8-6596921)

The Boussinesq system, arising as a result of the asymptotic expansion procedure applied to the Euler equations for the shallow water wave motion, is decomposed to a set of coupled equations for the right- and left-moving waves. It is shown that a non-uniqueness of such a decomposition can be used to derive a system, in which, to any order, one of the equations is dependent only on the main right-moving wave and has the form of the KdV equation with higher order corrections. As distinct from the unidirectional case, the coefficients in the higher-order correction differential polynomials are arbitrary. Some classes of solitary wave solutions of the right-moving wave equation, which in a sense include impacts of all orders of the asymptotic perturbation expansion, are constructed via a new approach to the use of the Lie-Backlund group transformations for differential equations dependent on a small parameter. The higher-order perturbed KdV equations, constructed in the course of application of the approach, are also considered in the context of asymptotic integrability. Those equations, on the one hand, are asymptotically integrable and, on the other, possess the alternative closed-form asymptotic solutions. Availability of two kinds of asymptotic solutions is used to examine to what extent the solutions derived via a near identity transformation from the normal form solitary waves are applicable to studying the high-order KdV solitons.