



## **Three-dimensional instabilities of periodic gravity waves in shallow water**

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Water-wave motion in coastal areas is a subject of prime importance to wave forecasting for meteorological and oceanographic purposes and ship navigation, etc.... Yet, the dynamics of water waves of large amplitude with typical wavelengths greater than the water depth are not well understood, although major practical problems are associated with surface coastal waves.

Theoretical models for shallow water waves often assume that nonlinear three-wave near-resonant and four-wave resonant interactions are the main nonlinear processes acting to transfer energy among the components of the wave field. It is well known that these interactions influence the growth rates of wind-generated gravity waves as well as other intrinsic properties of coherent surface gravity waves which can affect their own dynamics. Nevertheless, which wave-wave interactions dominate in shallow water remains unclear. Besides, it is well known that waves are usually more nonlinear in finite depth than the waves on deep water. Despite this, there appears that the higher-order resonant interactions problem has received little attention in finite depth and shallow water in comparison with the three-wave near resonant interactions problem.

In the present paper, we consider the linear stability analysis of finite-amplitude periodic surface waves propagating steadily on water of finite depth. This is a first step, in shallow water, towards the characterization of the instabilities resulting from higher-order resonances, i.e. between more than five waves. This study has extended existing results to steeper waves and shallower water. This also completes the previous work of Bryant (1974, 1978) and McLean (1982). Some new types of instability are found for shallow water. When the water depth decreases, higher-order resonances lead to the dominant instabilities. In contrast with the deep water case, we have found that in shallow water the dominant instabilities are usually associated with resonant interactions

between five, six, seven and eight waves. For small steepness, dominant instabilities are quasi two-dimensional. For moderate and large steepness, the dominant instabilities are three-dimensional and phased-locked with the unperturbed nonlinear wave. These results suggest, at the margin of instability diagram, the existence of bifurcated three-dimensional steady waves. These new waves possess few symmetry properties that are discussed in this paper.