



## **A new method for quantifying confidence region of normalised stress tensor**

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A variety of stress tensor inversion techniques have been proposed to deduce the stress fields in the earth's upper crust from fault-slip data. As well as methods to determine the optimal stress tensor itself, the statistical reliability of solution has been intensively investigated concerning with the effect of errors, biases and heterogeneity in the data. Up to the present, the reliability was often grasped in terms of principal orientations ( $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$ -axes) and stress ratios ( $\Phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ ). However, any assessment of stress tensor should not be based solely on orientations, since their stabilities depend on the stress ratio (Orife and Lisle, 2003), and vice versa.

For the purpose of evaluating the 'variance of tensor quantity', we modified the parameter space of Fry (1999) by differently weighting the diagonal and off-diagonal components of stress tensor. A six-dimensional vector corresponding to a stress tensor was named ' $\sigma$ -vector'. With the first and second invariants of stress tensor normalised to be zero and unity, respectively, the vector was found to lie on the five-dimensional unit sphere with four degrees of freedom. Furthermore, the Euclidean metric of our parameter space has a physical meaning of the stress difference (Orife and Lisle, 2003), which is a proper measure of difference between normalised stress tensors. Consequently, the covariance matrix of  $\sigma$ -vectors gives a straightforward measure of variance of their mean stress tensor.

Practically, we utilised the bootstrap resampling method to scatter the  $\sigma$ -vectors. The method produced a great number of datasets by iteratively sampling faults from the original dataset. For each dataset, the optimal solution was determined by solving eigenvalue problem (Fry, 1999). The resultant  $\sigma$ -vectors were orthogonally projected onto the plane which was tangent to the five-dimensional unit sphere at the point indicated by their mean vector. Assuming that the vectors obeyed the four-dimensional

normal distribution on the plane, we could calculate the covariance matrix and defined the confidence region as four-dimensional ellipsoid. The paired stereograms of Yamaji (2000) enabled the visualisation of confidence regions. As case studies, artificial and natural fault-slip data were analysed. The quantified confidence regions of their optimal solutions demonstrated the dependence of stability of principal orientations on the values of stress ratios.

#### References

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