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Calculating covariances of random functions linked by poisson equation in $\mathbf{R}^2$

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The application of Geostatistics to Hydrogeology, and more generally to physical phenomena governed by Partial Differential Equations (PDE), requests the explicit knowledge of the spatial covariance or variogram models for the Random Functions (RF) which represent the different physical quantities. In fact, for linear relationships, the equation of the phenomenon is interpreted in stochastic sense and converted in differential equations between simple and cross-covariances (or variograms) of the RFs. According to theorems by Matheron ([1]), the different RFs do not have the same degree of stationarity. However, due to their complexity, is not easy to integrate these functional relations, so that it is often necessary to simplify the border conditions or to recur to a numerical approach ([2], [3]).

In this work it is shown that the classical linearized form of the Diffusivity Equation, linking the perturbations of hydraulic head and transmissivity in groundwater under stationary parallel flow conditions, leads to functional relations between covariances which are analitically and directly solved using the single hypothesis of isotropy (in geostatistical sense) of the transmissivity perturbation. The solution consists in transforming the original differential equation (on R^2 or R^3) into an iterated integral in one dimension. This method, completely general, appears to be applyable to a wide class of PDEs, therefore extending the family and the complexity of physical phenomena approachable by Geostatistics.

Finally, relations between covariances of hydraulic head and transmissivity are obtained also in the case of different macroscopic flow conditions (e.g. winter/sum-mer), which turns out to be more realistic for a greater number of real cases; this is particu-

larly useful in the perspective of inverse problem, in order to calculate transmissivity fields from head measurements.

References

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