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Extremes of Multifractal Cascades: Exact Distribution and Approximations

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In many applications, one is interested in the maximum M of a multifractal measure at a given resolution r. Previous studies have focused on the scaling of quantiles of M with the exceedance probability and the resolution, but this falls short of characterizing the distribution F_M. While calculation of F_M for continuous multifractal models is difficult, in the case of discrete cascades such distribution can be obtained through an iterative numerical procedure. The distribution depends on the cascade generator Y, the integer multiplicity of the cascade m, and the resolution $r = m^n$ at which the measure is considered. We evaluate F_M for lognormal and beta-lognormal cascades and study its sensitivity to simplifying approximations, including the assumption of independence of the measure in different cascade tiles and the replacement of the dressing factor by a random variable of the same type as the generator Y. We also examine how F_M varies with the multiplicity m and the Euclidean dimension of the support d.

Concerning dependence among the cascade tiles, we find that: (i) at high resolutions r, dependence affects significantly the body and lower tail of F_M but not the extreme upper tail; (ii) for low r, the entire distribution F_M is unaffected by dependence; (iii) long-range dependence is more important for F_M than short-range dependence; and (iv) for log-stable cascades with fixed index of stability α, the effect of dependence on F_M varies in a simple analytic way with the co-dimension parameter C_1. We also find that approximating the dressing factor with a variable having the same distribution type as Y induces little error on the distribution of M, except in the extreme upper tail region. The effect of m on F_M is modest. Also the effect of the Euclidean space dimension d is small, if what is kept fixed is the volumetric (not the linear) resolution.

We use these findings to propose a simple approximation to the distribution of M that includes the effect of dependence, and give charts and explicit formulas to implement the approximation for beta-lognormal cascades.