



Direct numerical simulation of evolution of random water wave fields: direct and inverse cascades

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The aim of the work is to check by direct numerical simulations (DNS) the validity of the classical solutions of the spectral transfer equation in a number of model situations, to validate the hypotheses underlying the statistical theory of surface waves, and to get solutions for the cases where this theory is not applicable. We employ a particular DNS approach based on the integrodifferential Zakharov equation for simulation of long term evolution of random water wave fields. The numerical method includes building of a dense grid (with the number of harmonics $\sim O(10^6)$) of waves coupled by resonance interactions, with subsequent grouping into a smaller ($O(10^3)$) number of wave packets, so that each packet has the amplitude of the envelope of harmonics it comprises and a randomly chosen phase, with all the resonant interactions of the original harmonics taken into account. The simulations were carried out for two types of situations: initial conditions were either a narrow high-frequency peak, or white noise, with pumping to high-frequency part of the spectrum. Numerical solutions are compared with the results obtained with the spectral transfer equation.

In the initial part of evolution ($\sim 10^2$ characteristic periods), the evolution timescale is shown to be much smaller and to depend linearly on the field energy, contrary to the quadratic dependence predicted by the statistical theory. At this stage, the evolution of spectra obtained by means of DNS is considerably faster and, from the viewpoint of the spectral transfer equation, represents an effective modification of its initial conditions. Formation of powerlike spectra corresponding to direct cascade of energy to high frequencies is demonstrated. At a later stage of the evolution ($10^3 - 10^6$ periods), well pronounced powerlike spectra corresponding to inverse cascade of wave action are formed. Computations with higher-order interactions taken into account reveal a certain distortion of these spectra for large times. Evolution of higher-order statistical

momenta is also discussed.

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