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Compound Poisson-multifractal processes and rain modelling from drop scales on up

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Many scaling phenomena are fundamentally pointlike but when averaged over larger enough scales are considered to continuous and mathematically modeled using fields (or densities of measures); examples range from the large scale distribution of matter in the universe, to seismic events to raindrops. By studying the example of rain we show how such systems can be modeled using compound Poisson/multifractal processes, the discussion will be particularly concrete since on the one hand there are now direct (stereophotographic) reconstructions of three-dimensional distributions of rain drops (the HYDROP experiment), and on the other elements of classical turbulence theory can be used to make the whole model grounded in solid turbulence theory.

The traditional picture of rain is that at large scales, it is a field; a liquid water density (ρp multiplied by a velocity; both fields being subject to turbulence. At small scales, rain is particulate, the drop size distribution being determined largely by inter-drop collisions/interactions. Traditionally, the transition from particle to field descriptions is made by assuming the continuum hypothesis: i.e. a macroscopic/microscopic scale separation. ON the contrary, the HYDROP experiment showed that when ρ is averaged over larger and larger spheres, it is found (in accord with turbulence theory) that it tends to a heterogeneous (not homogeneous), hierarchical, multifractal clustering limit. The scale at which this (scaling) clustering starts depends in a theoretically predictable way on the intensity of the turbulence and on the drop size distribution, but was typically 30 -50cm. In addition, spectral analysis of the larger scales directly provides a link to classical turbulence theory by showing that the energy E(k) spectrum

of ρ is very close to the k$^{-5/3}$ Corrsin-Obhukov form predicted for passive scalars (k is a wavenumber).

We outline a new model of rain valid from huge (turbulence dominated) scales down to individual drop scales. The key is the ρ variance flux (χ) which - following the HYDROP observations and Corrsin-Obukhov theory – is conserved from scale to scale it is the basic multifractal field. The link to the particle description is via the particle number density (n); we show how this can be determined from χ and the energy flux ε; we predict a k^{-2} spectrum for n which we confirm is close to observations.

In order to perform simulations respecting these turbulence constraints we start with multifractal models of chi and epsilon cut-off by viscosity at the dissipation scale (e.g. 1cm). From these fluxes we determine ρ and n by fractional integration. At scales below 10cm or so, there is typically only one drop in the corresponding sphere; we interpret n as the number density of a (compound) Poisson process and randomly determine the positions of the ith particle: r_i. The masses m$_i$, are determined from a unit exponential random variable E$_i$: m$_i$=E$_i\rho$or$_i$p/nor$_i$p. The resulting measure (m$_i$, r$_i$) has the observed energy spectrum, the observed multifractal statistics (including the transition from particle scales to field scales) it also has realistic probability (fat tailed, power law) distributions for m. Since it incorporates (in a highly inhomogeneous framework) the Marshall-Palmer exponential drop distribution as well as a Poisson particle process, it bridges the gap between classical and turbulence approaches.

Numerical simulations spanning the range 1cm to 1000km can be readily produced. These simulations can be used for simulating radar reflectivity factors, effective radar reflectivity factors; extensions of the model can be used to simulate rain rates and rain gauges. These models can thus potentially solve various precipitation observer problems.