



A Large, Spurious Source of Energy in Mantle Convection: Specific Heat, the Equation of State, and their Effect on Energy Conservation

C. F. Hieronymus

ETH Zurich, Institute of Geophysics (hieronymus@erdw.ethz.ch)

The specific heat describes the temperature change of a unit mass of material resulting from a given heat input. The specific heat depends on the external conditions under which a thermodynamic process takes place. Specifically, when the ambient pressure is non-zero, mechanical work is done as the material thermally expands (or contracts). As a result, a given heat input at high pressure is partitioned into raising the temperature of the material and into mechanical work associated with volumetric expansion against pressure. The heat capacity is thus generally higher at high pressure. In small systems at near-atmospheric pressure, the mechanical work can usually be neglected. In planetary-scale convection where the hydrostatic pressure exhibits large variations, the specific heat at depth is significantly greater than at the surface; for the Earth's mantle, the correction is around 10-25%. When material is heated at depth, the mechanical work is transformed into gravitational potential energy as the overlying column of material is lifted up. This sink of thermal energy approximately balances the heat generated by viscous dissipation as cold material sinks and hot material rises through the mantle. While the uplift of material caused by volume changes can usually be neglected in convection models, the corresponding loss of thermal energy needs to be properly accounted for. Without this thermal energy sink, shear heating leads to a spurious and continuous source of net energy which amounts to about 10-25% of the total energy budget of the Earth. In larger planets with a deeper convecting interior, the contribution of the spurious energy source may be greater than the instantaneous heat loss through the surface. Models with constant heat capacity and with shear heating as the only energy source then result in a (clearly unphysical) perpetual motion machine. Depending on the material's equation of state, a constant specific heat as commonly used in convection models is thus incompatible with energy conservation. The only

exceptions are equations of state of the same form as the ideal gas law which can be shown to have a specific heat that is independent of pressure.