



Variational methods for ice stream flow over a plastic bed

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Experimental work conducted over the last decade has provided a wealth of evidence for the approximately Coulomb-plastic behaviour of subglacial tills. If the sliding motion of glaciers and ice streams is largely the results of deformation within these tills, then the appropriate sliding law for these ice masses is simply a Coulomb friction law in which basal shear stress is independent of sliding velocity. The same is true if differential motion described by a Coulomb friction law occurs at the ice-till interface (Tulaczyk, 1999) rather than within the till. The incorporation of Coulomb-frictional sliding into large-scale ice sheet models is difficult because most of these models are based on the shallow ice approximation, which assumes in its simplest form that vertical shear stresses are large compared with lateral or longitudinal stresses, and that sliding velocity can be computed as a function of gravitational driving stress. In regions where Coulomb-frictional sliding occurs, this does not hold true, and longitudinal and lateral shear stresses are the dominant components in force balance (Joughin and others, 2004). This leads to the conclusion that parts of an ice sheet in which basal shear stress is below the yield stress of the bed, and in which there is little or no sliding, need to be described by a different mathematical model from those in which the yield stress of the bed is attained. A further problem now arises: How do we know where the yield stress is and is not attained? The question is moot for present-day ice sheets, where velocity maps can be used to infer where rapid sliding is taking place. However, in a predictive ice sheet model, these regions should be able to evolve (ice streams can widen, narrow, switch on or off), and determining the spatial extent of ice stream flow becomes a free boundary problem (Schoof, in press). In this presentation, we show how variational methods can be used to solve this free boundary problem in a very succinct way, and discuss the challenges our method faces when applied to a

dynamic ice sheet model.

References

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