Geophysical Research Abstracts, Vol. 7, 02895, 2005
SRef-ID: 1607-7962/gra/EGU05-A-02895


# Modeling oceanic mixing by stochastic flows

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Numerous observations in the ocean show that the velocity fluctuations of drifters are well described by linear stochastic differential equations of the first order, in the simplest case by the Langevin equation. Based on this fact, a general stochastic model has been suggested and developed by the author to describe multi particle motion as well as evolution of a continuously distributed tracer in the presence of eddy turbulence. A simplified version of that model in the $2D$ case with velocity homogeneous fluctuations is given by

$$d\mathbf{r} = \mathbf{v}dt, \quad d\mathbf{v} = -(\mathbf{v}/\tau)dt + d\mathbf{w}(t, \mathbf{r}), \tag{1}$$

where $\mathbf{v}, \mathbf{r}$ are the velocity and position of a particle, $\tau$ is the Lagrangian correlation time and $\mathbf{w}(t, \mathbf{r})$ is a Brownian motion in a Hilbert space.

One of the most important mixing parameters is the top Lyapunov exponent, $\lambda$. We show that for flow (1) it is given by

$$\begin{array}{c} \lambda = |D|^{1/3}(-\sqrt{c} - Ai'(c)/Ai(c)) \text{ if } \lambda_B > 0 \\ \text{and } \lambda = |D|^{1/3}(-\sqrt{c} + Bi'(c)/Bi(c)) \text{ if } \lambda_B < 0 \end{array} \tag{2}$$

where $c = D^{-2/3}\tau^{-2}, \quad D = \lambda_B \tau^{-2}$, $\lambda_B$ is the top Lyapunov exponent for the Brownian flow

$$d\mathbf{r} = \tau d\mathbf{w}(t, \mathbf{r}) , \tag{3}$$

and $Ai, Bi$ are Airy functions. It follows from (2) that if $\lambda_B > 0$, then $\lambda > 0$ as well, however in the opposite case , $\lambda_B < 0$, $\lambda$ can be either, negative or positive. Notice that $\lambda_B = \sigma_u^2 \tau / R^2$, where $\sigma_u^2$ and $R$ are the variance and correlation scale of the velocity from (1).

Next, we investigate the relative dispersion: both, the Brownian flow (3) and the first order Markov flow (1) have sufficient capacity for modeling known physical regimes including Richardson $t^3$. However, the inertial particle model (1) is certainly more preferable for mimicking real turbulence since it is able to reproduce the ballistic regime and gives a more realistic dependence of the relative dispersion slope on the scaling of flow structure function.

Finally, we present preliminary numerical results on the continuously distributed tracer stirred by (1).

The author greatly appreciate the support of Office of Naval Research under grant N00014-99-1-0042.