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Local Lyapunov Exponents

W. Siegert , P. Imkeller
Humboldt University Berlin (siegert@math.hu-berlin.de , imkeller@math.hu-berlin.de)

In this talk we investigate the nonlinear stochastic differential equation

$$\begin{aligned} dX_t^\varepsilon &= -U'\left(X_t^\varepsilon\right) dt + \sqrt{\varepsilon}\, dW_t \qquad (t \geq 0\,,\ \varepsilon > 0)\,, \\ X_0^\varepsilon &= x_0 \in \mathbb{R}\ , \end{aligned}$$

where U is a double-well potential with two minima of different height and W is a one-dimensional Brownian Motion. Linearizing the random dynamical system X^ε along trajectories, one obtains the linear system V^ε associated with the (one-dimensional) variational differential equation

$$\frac{dV_t^\varepsilon}{dt} = -U''\left(X_t^\varepsilon\right)\, V_t^\varepsilon\ ,$$

which is used to study the long-term behavior of the nonlinear system X^ε. For the linearization V^ε there is one *Lyapunov exponent* λ^ε, i.e. almost sure exponential growth rate as $t \to \infty$, according to Oseledets' Multiplicative Ergodic Theorem (which here in the one-dimensional case reduces to Birkhoff's ergodic theorem). Due to ergodicity, λ^ε does not depend on the initial value x_0.

The goal of this contribution is to provide a Lyapunov-type characterisic number for each well of the potential. As this number shall depend on x_0 (more precisely on the well in which X^ε is starting), it yields a concept of *Locality* for Lyapunov Exponents. Up to now local Lyapunov Exponents have been defined as finite-time versions by several authors; in our work we chose to stick to the asymptotics $t \to \infty$. The main motivation is that in the small-noise-limit the particle X^ε stays in the shallow well for an exponentially long time (Kramer's law, Freidlin-Wentzell theory) during which we can "see" the shallow well; afterwards the deep well dominates the picture, as the invariant measure has its maximum there. So we connect the large parameters t

and ε^{-1} in the definition of the Lyapunov exponent in order to approach the *sublimit distributions* (Freidlin). As a result, the local Lyapunov exponents are given by the curvature $-U''$, evaluated at the minima of the two potential wells, and depend on x_0 as well as on the time scale chosen; the latter fact reflects the metastability of the system.