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SHARP-CRESTED GRAVITY WAVES WITH ANGLES OTHER THAN 120°

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In the framework of the canonical model of hydrodynamics, where fluid is assumed to be ideal and incompressible, waves are potential, two-dimensional, and symmetric, the authors have recently provided numerical evidence for the possible existence of sharp-crested gravity waves other than a 120° limiting Stokes wave [1, 2]. To clarify these recent results, the purpose of this presentation is to show that the canonical model admits sharp-crested solutions with crest angles other than 120°. To this end, we generalized the well-known Michell's method (which is usually used to calculate a 120° Stokes corner flow on deep water) to include an arbitrary angle at the crest. The complex velocity of a corner flow with angle $\alpha = \pi(1-\nu)$ is described as

$$\frac{\mathrm{d}w}{\mathrm{d}\zeta} = -c\,(1-u)^\nu \sum_{n=0}^{N} b_n\, u^n, \quad u = \exp(\mathrm{i}\, w/c), \quad b_0 \equiv 1,$$

where w is the complex potential, $\zeta = x + \mathrm{i}\, y$ (x and y being the horizontal and vertical coordinates with origin at the wave crest), and c is the wave phase speed (waves propagate along the x-axis). Substitution of this expansion into the Bernoulli equation leads to a set of non-linear algebraic equations for the real coefficients b_n ($n = \overline{1,\, N}$) and the wave phase speed c. When only the first term ($n = 0$) of the expansion is considered, the Bernoulli equation admits no other possibility than $\nu = \frac{1}{3}$, i.e., $\alpha = 120^\circ$ (famous Stokes theorem). In view of this, original Michell's method incorporates the *a priori* assignment $\nu = \frac{1}{3}$. We show that for $N > 0$ the condition $\nu = \frac{1}{3}$ is not necessary and the Bernoulli equation generally admits solutions with $0 < \nu < \frac{2}{3}$ ($180^\circ > \alpha > 60^\circ$). Numerical results demonstrate that the waves with $\alpha > 120^\circ\, /\, \alpha < 120^\circ$ have smaller / higher amplitudes (trough-to-crest heights) as compared to the limiting Stokes wave.

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[2] Lukomsky V. P., Gandzha I. S., Lukomsky D. V. Reply Comment on "Steep sharp-crested gravity waves on deep water" // Phys. Rev. Lett. – 2004. – Vol. 93, No. 6. – P. 069 403–1.