# Geostatistical interpolation of runoff

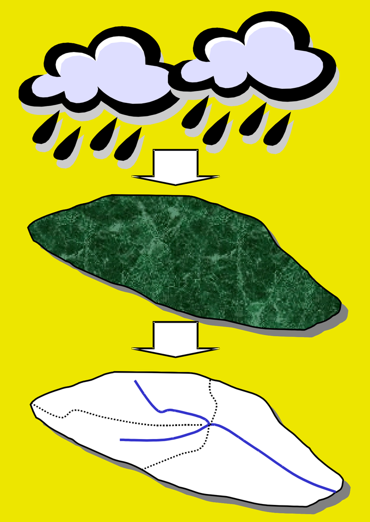
Jon Olav Skøien and Günter Blöschl
Institute of Hydraulic and Water Resources Engineering
Vienna University of Technology

## Motivation

- Common practice to use deterministic models for prediction of runoff in hydrology
- Stochastic models have been used for regionalisation of model parameters

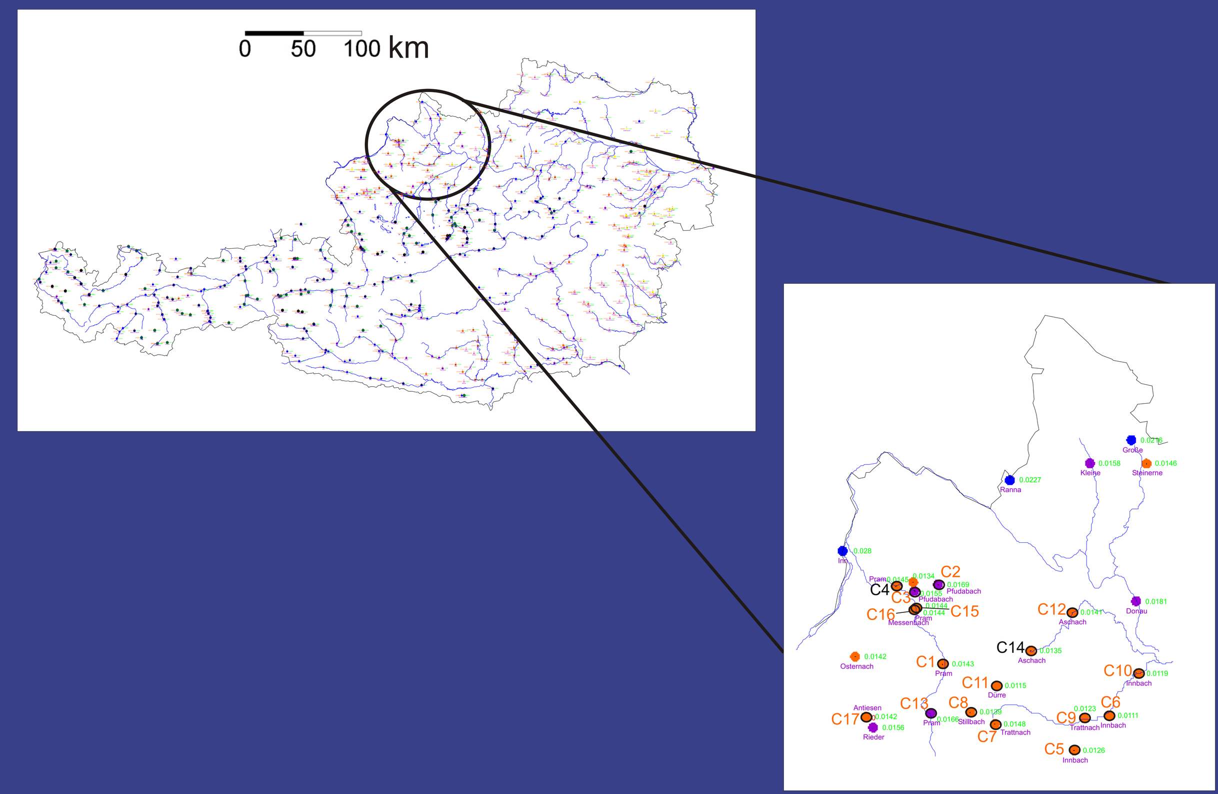
➡ We suggest a geostatistical interpolation method for runoff taking into account:
- Runoff from neighbouring catchments
- True catchment boundaries
- Network structure

| | Type of variability | Our approach |
|---|---|---|
| Atmospheric forcing | Euclidian | Variogram |
| Soil and vegetation | Euclidian | Variogram |
| Channel network | Tree structure | ➡ Boundaries |

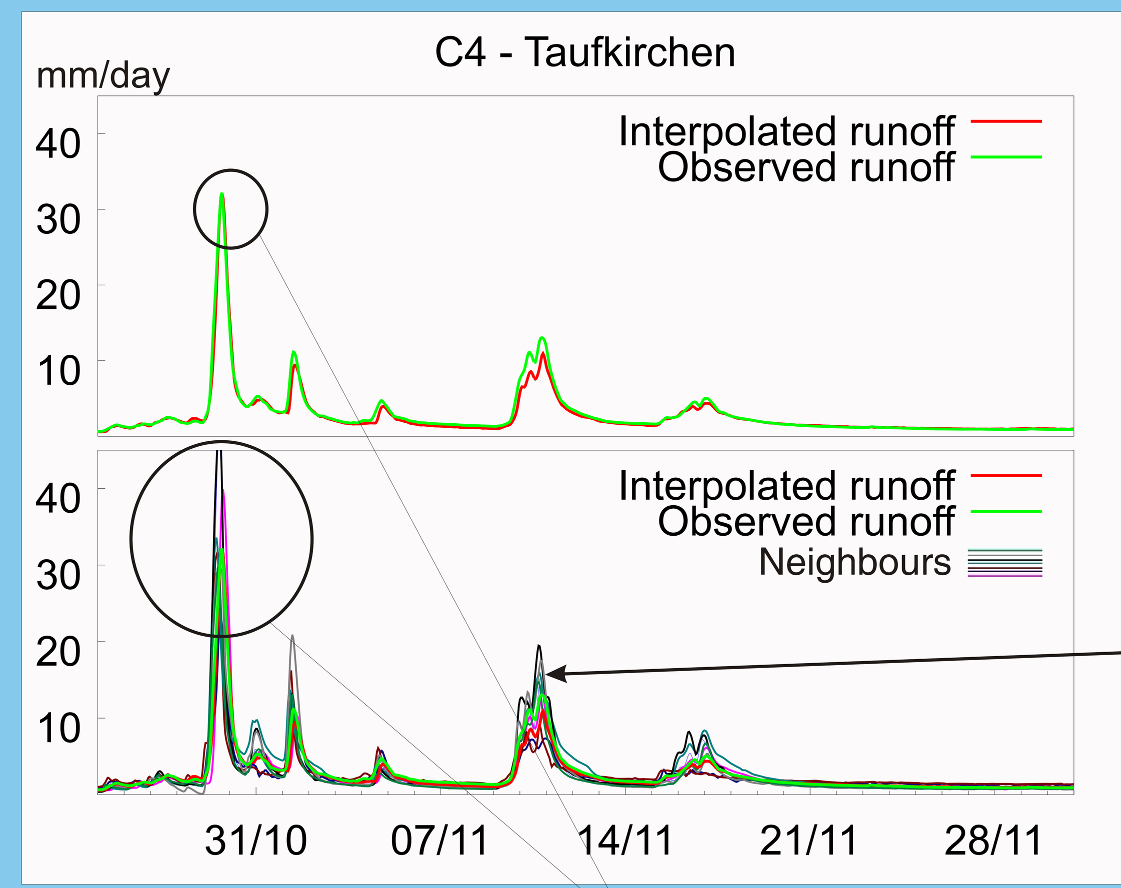


## Data

- Interpolation of 17 stations from Innviertel, Austria
  - marked with black circles, numbered C1-C17
- Yearly runoff relatively homogenous (small green numbers)
- Also other neighbouring catchments used for interpolation




## Results

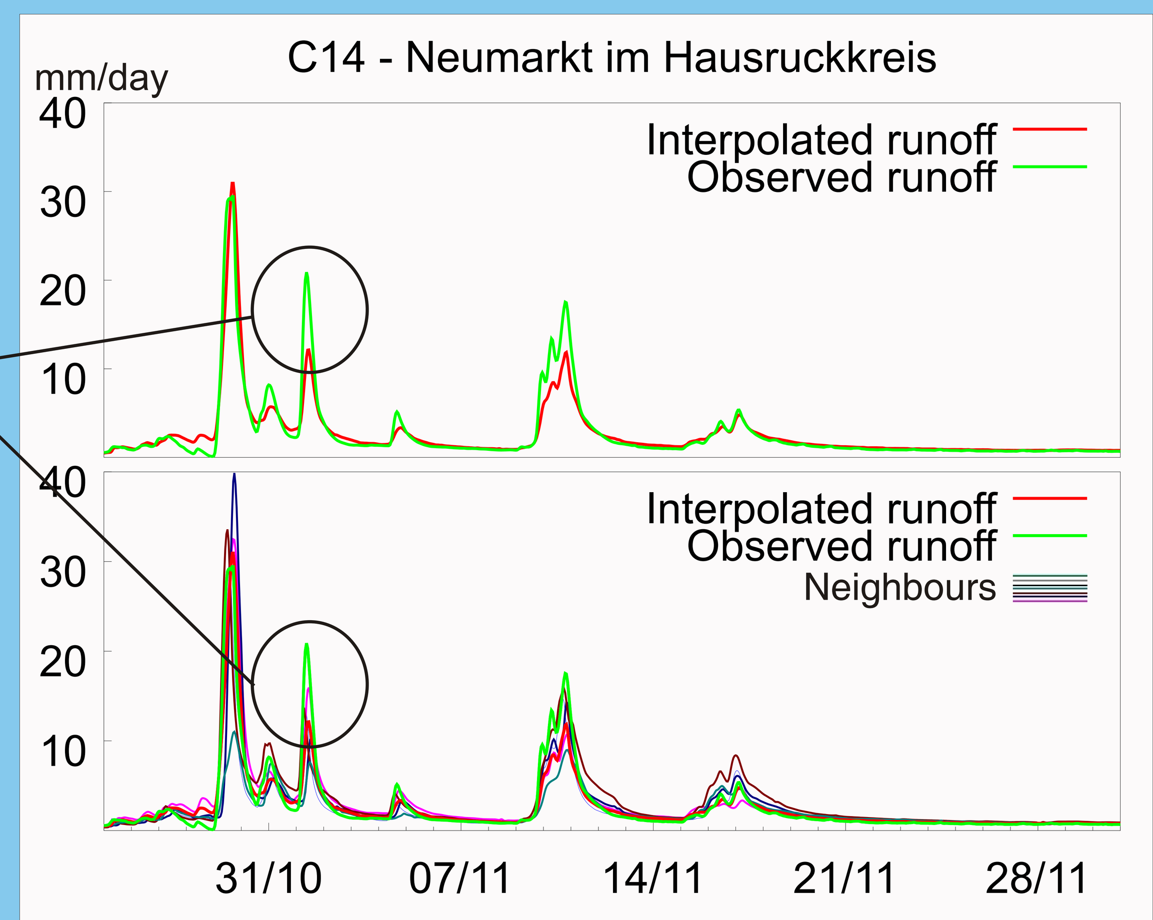



**Two interpolated runoff hydrographs as examples**
- Hourly runoff 1998 interpolated
- Only showing 35 day period
- Nash-Sutcliffe for all catchments.0.75-0.97

Hydrographs from some of the neighbours used for interpolation

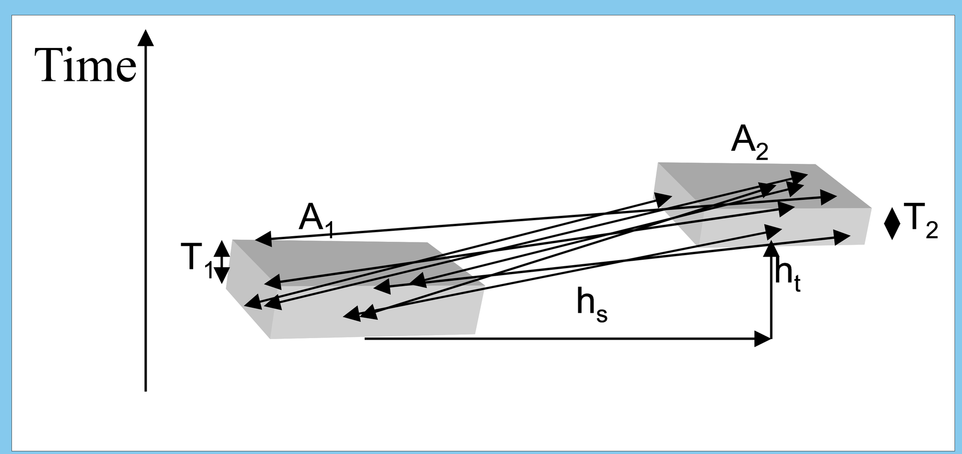
**Peak satisfying interpolated despite large variance in peaks of neighbouring catchments**

**Second peak largest in interpolated catchment => underestimation**
- Method is not able to preserve variance of a time series
- Kriging gives the best linear unbiased estimator for each time step




## Estimating variance between catchments

- The catchment filters the signal from the atmospheric forcing
  - The smoothing is stronger from a large catchment than from a small one
- Also temporal filtering takes place, due to the residence time in the catchment
- The residence time simplified as $T=$ $A$ , $A$ is area and and parameters
- We assume a simple aggregation within the catchment
  - Runoff at outlet is average of runoff generated within the catchment during time T




**Catchment: can be seen as a set of points in space and time**
- Variance between catchments
  - The average of point variances
- Variances dependent on spatial and temporal distances

➡ Regularisation

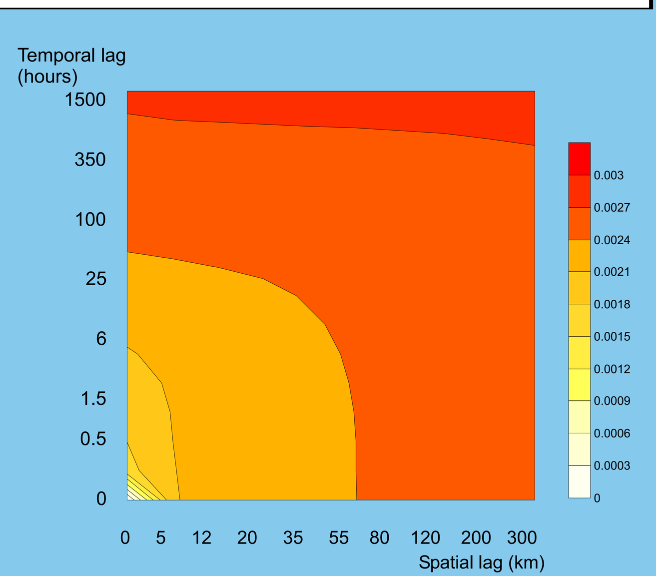
**Regularisation of a variogram in space and time:**

$$_{st}(h_s|a,h_t|T) \quad \frac{1}{A^2T^2}\int_{ATAT} (|s \quad h_s \quad r|,|t \quad h_t \quad |)dsdrdtd \quad \frac{1}{A^2T^2}\int_{ATAT} (|s \quad r|,|t \quad |)dsdrdtd$$

**Back-calculation of a point variogram:**

Spatio-temporal sample variogram of runoff
- Variogram cloud of the catchment pairs
- Binned temporal distances

=>Point variogram found by joint fitting of the spatio-temporal variances between catchments




Spatio-temporal point variogram of runoff

## Block Kriging

- Gamma value in kriging matrix calculated separately for each pair of catchments
- Kriging matrix solved as normally
- The kriging matrix is only solved once for each ungauged catchment.

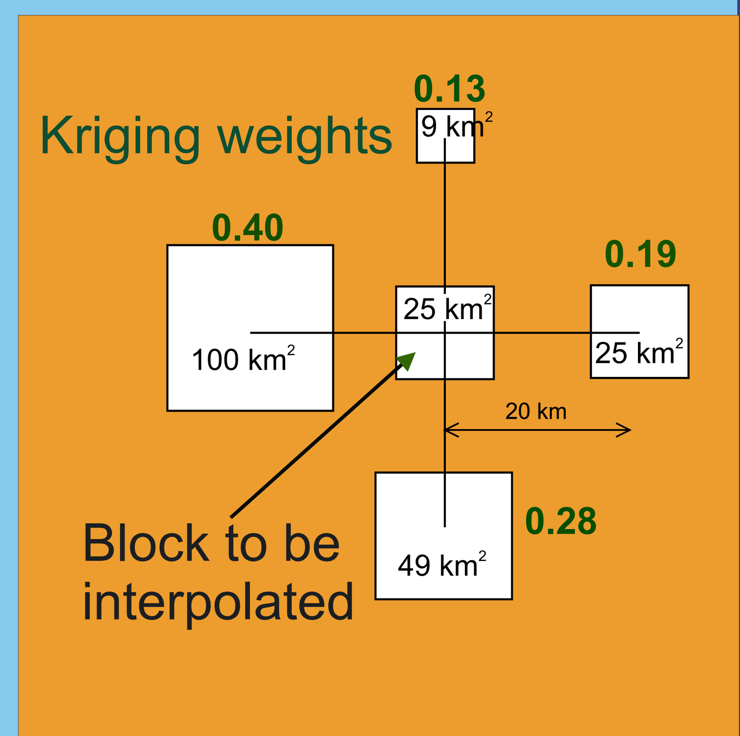
$$_{ij} \quad \frac{1}{A_iA_jT_iT_j}\int_{A_iT_iA_jT_j} (s \quad r,t \quad )dsdrdtd$$

$$0.5 \quad \frac{1}{A_i^2T_i^2}\int_{A_iT_iA_iT_i} (s \quad r,t \quad )dsdrdtd \quad \frac{1}{A_j^2T_j^2}\int_{A_jT_jA_jT_j} (s \quad r,t \quad )dsdrdtd$$

## The influence of support on interpolation

**A spatial example - Four neighbour blocks with equal centre-to-centre distance**

- Largest catchment will get largest kriging weight
- Smallest catchment get smallest weight
- Size of interpolated block does not come into account in symmetric case




## Conclusion

**Spatio-temporal geostatistical interpolation offers an alternative to regionalisation of rainfall-runoff model parameters for ungauged catchments (PUB)**
- Method uses real catchment boundaries
- No information in addition to runoff data and catchment boundaries needed
- To do:
  - Consider other aggregation schemes
  - Preserve variance in a better way

EGU General Assembly
Vienna, April 25-29, 2005
Contact: Jon Olav Skøien
skoien@hydro.tuwien.ac.at